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Branching model for the fracture of fibre composites

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Models of localized and delocalized fracture of fibre reinforced composite materials have been considered from the viewpoint of the theory of branching processes. The analysis has shown that, in spite of apparent differences, both types of models can be reduced to generally the same Markov chain. As a result, a new fracture criterion has been proposed that is valid for any model. The use of the new criterion allowed for the revelation of a new structural effect, i.e. the dependence of the fracture stress of the composite upon the size of the cross-section of the composite sample. In the case of a fracture of an infinitely large composite sample, the criterion yields the same fracture stress as calculated on the basis of earlier models. In the case of the fracture of a sample of a finite size, the predicted fracture stress is lower than calculated according to previous models. The effect can be explained as a non-linear fracture phenomenon arising out of the non-linear dependence of microfracture probabilities upon overstressing caused by other microfractures. The effect is essential for evaluating the strength of a structured composite with several levels of ordering and constriction elements of a small size. © *1998 Chapman & Hall*

1. Introduction

In general, publications devoted to the mechanical properties of composite materials consisting of a ductile matrix reinforced with continuous high-strength high-stiffness fibres focus on two types of fracture models. Models of one type can be called models of delocalized fracture, whereas models of the other type can be characterized as ones of localized fracture. A complete theory of a composite fracture should take into account both fibre fractures and non-fibre effects, such as matrix plastic flow, debonding, residual stresses, disordering of a regular structure, etc. These effects have different contributions to a fracture pattern and in many cases can be accounted for by superimposing models developed for each particular fracture mechanism. The present paper considers statistical aspects of composite fracture conditioned by fibre breaks.

The model of delocalized fracture was first introduced by Rosen [1]. This model is based on the mechanism of the failure of a bundle of fibres having a statistical distribution of flaws or imperfections and has been thoroughly considered by Daniels [2]. According to the suggested approach, a fibre is considered as a chain composed of links of length d, which is the ineffective length of a fibre in the matrix. The links have a satistical distribution of strength due to imperfections. It is also supported that each link can be broken only once when tension is applied.

A composite as a whole can then be considered as composed of a series of identical layers of dimension d. Each layer in this case represents itself as a bundle of fibre links and the composite is treated as a series of such bundles. Any fibre that fractures within the layers is supposed to be unable to transmit a load across the layer and the applied load is treated as uniformly distributed among the unbroken fibres in each layer. (For simplicity, it is assumed that only fibres can carry the tension, whereas the matrix transmits the shear stress.)

From the assumptions of the model, it follows that with an increase of the applied load fibres fracture in random places within the material and composite failure occurs due to the statistical accumulation of flaws (fibre breaks) in a layer.

The concept of localized fracture was first considered by Zweben [3]. For the description of a composite structure, basically the same model assumptions were used. The composite was considered to consist of a series of identical layers of elements whose axial dimensions were some ineffective length d. Any fibre that broke in a layer was considered to have zero stress within that layer, but to be fully stressed in all other layers. However, in contrast to Rosen, the model assumed that elements adjacent to a broken one were subjected to a load intensity greater than that which was sustained by fibres distant from the fracture site. Therefore, each fracture site could be thought of as a nucleus for the propagation of fibre fracture, and in this case fibre breaks have to have a tendency of localization. The localization means that with the increase of the applied load new fibre breaks occur predominantly in the vicinity of already existing fibre breaks. The composite fracture in this model is associated with the growth of a flaw due to successive breaks of neighbouring fibres.

Since their first publication both models have been studied many times in regard to theoretical elaboration as well as in comparison with experimental data. The most complicated case was considered in [4], where different types of load-sharing rules were investigated. Also, the case where fibres located at different distances from a fracture site experience different degrees of overstressing has been analysed. It was shown that the fibres under maximal overstressing play a key role in fracture development.

The delocalized and localized fracture models use different criteria for a composite fracture (macrofracture). The purpose of the present study is to introduce a common fracture criterion applicable both to delocalized and localized fracture models. The use of a common criterion will allow us to understand better the nature of discrepancies between the models (if any) and to determine in different cases which model is more appropriate and also to reveal a new structural effect that has not been described by earlier models.

The criterion to be used comes from the theory of branching processes and is different from criteria used earlier. In order to introduce a new criterion, we will start by comparing it to criteria that have already been used.

2. Modelling

2.1. Delocalized fracture model

2.1.1. Basic theory

For the purpose of further comparison this section summarizes the main features of the delocalized fracture model described in [1]. Consider one layer of a composite (the layer is treated as a bundle of fibre links). Due to basic model assumptions the fracture of a composite can be reduced to a fracture of a separate layer (the weakest layer). A composite itself is assumed to have been destroyed when the weakest layer is fractured. Let the initial number of unbroken fibres in the layer be N_0, and the cumulative distribution function, $F(s)$, of the fibres' strength in a layer is described by the Weibull distribution

$$F(s) = 1 - \exp[-a(s/s_0)^b] \tag{1}$$

where a, s_0 and b are the distribution parameters, and s is the acting stress. It is supposed that each fibre in the layer can be broken only once as the load applied to the composite increases.

The total load carried by the layer, P, is related to the stress acting in unbroken fibres by the equation

$$P(s) = Ns = N_0 s[1 - F(s)] \tag{2}$$

where N is the current number of unbroken fibres in the layer. (Here, for simplicity, each fibre cross-section is taken to be one unit of area.)

With the increase of the acting stress, the number of undamaged fibres decreases; therefore, P as a function of s multiplied by N has a maximum. The stress, s_c, at which this maximum is reached can be evaluated from the equation

$$\mathrm{d}P(s)/\mathrm{d}s = 0 \quad \text{at} \quad s = s_c \tag{3}$$

The value s_c is usually taken as the most probable failure stress and Equation 3 is considered as the fracture criterion in models of a delocalized fracture.

Using $F(s)$ in the form of Equation 1 one can obtain from Equation 3

$$s_c = s_0(ab)^{-1/b} \tag{4}$$

Consider the same model of a delocalized fracture from the viewpoint of the theory of branching processes. Let the initial model assumptions be the same, i.e. the composite is considered as consisting of a series of identical layers of a certain length, d, undamaged fibres are equally stressed within a layer, and broken fibres carry no stress within that layer where the break is but sustain a total load in other layers. Nevertheless, in contrast to previous considerations, we will analyse the processes related to fibre break in more detail.

Whenever a fibre is broken the stress equilibrium in the layer is disturbed and then it starts recovering into a new state of equilibrium. The recovery occurs in several steps. After a fibre break, the load which that fibre carried is then equally redistributed between the other fibres in the layer. The additional overstressing is $\Delta s = s/(N - 1)$. As a result of this overstressing, additional fibres can be broken. The number of those additionally broken fibres, N_1, is equal to the number of fibres whose strength is within the range $[s; s + \Delta s]$, i.e.

$$\begin{aligned} N_1(s) &= N_0[1 - F(S)] - N_0[1 - F(s + \Delta s)] \\ &= N_0[F(s + \Delta s) - F(s)] \end{aligned} \tag{5}$$

The rupture of N_1-fibres causes new additional overstressing, i.e. $\Delta s_1 = (s + \Delta s)/(N - N_1 - 1)$. This overstressing will cause new fibre breaks, of total number N_2; the breaks of N_2 fibres will cause new additional overstressing and so on until some equilibrium state is reached.

The process of successive fibre breaks described above is a typical branching process and, therefore, we can use the theory of branching processes for further consideration.

From the viewpoint of branching processes, N_1 fibres, broken as a result of the initial fibre break, can be seen as the first generation of a branching process. N_2 fibres, broken as the consequence of N_1 fibre breaks, will constitute the second generation of the process, etc.

For convenience, we call the process of fibre breaks induced by another fibre break as a correlated process of fibre breaks. This is in order to distinguish such a process from the process of independent fibre breaks caused by an increment of the applied load.

The total number of fibres injured due to initial fibre break at stress s, $N_r(s)$ is

$$N_r(s) = \sum_{k=1}^{\infty} N_k(s) \tag{6}$$

Here $N_k(s)$ is the number of fibres broken in the kth generation of a process of correlated fibre breaks. Note that $N_k(s)$ are random numbers, characterized by their average and dispersion.

One can see that if $N_r(s)$ is finite, that means that the process of correlated fibre breaks will eventually become extinct. The extinction, in turn, means that a new state of equilibrium is reached in the layer at applied stress s. At the same time, whenever $N_r(s)$ goes to infinity, this means that all fibres in the layer will be eventually destroyed at that stress, s, due to a correlated process of fibre breaks. This destruction means that macrofracture of the material has occurred, and, therefore, the condition $N_r(s) \to \infty$ can be considered as a condition for determining a composite fracture stress, and the described branching process can be employed for formulating the fracture criterion.

Compare the critical stress determined by the process of correlated fibre breaks with the critical stress determined by Rosen's model of delocalized fracture. In the general case, calculating $N_r(s)$ and the critical stress following from the condition $N_r(s) \to \infty$ requires quite elaborate mathematical calculation, but for the purpose of quick evaluation, we can obtain an approximate solution, which tends to be precise, in a limiting case.

Now we will make an assumption, which simplifies things, in order to obtain the analytical expressions for the fracture criterion. (Later on we will consider how essential the influence of the assumption is on the final result.) Because in the case of extinction $N_k \ll N_0$, we can take $\Delta s = \Delta s_1 = \Delta s_2 = \cdots = \Delta s_k$. In this case the branching process can be considered as a regular Markov chain. In the case when Δs_k is different, a consideration similar to the following can be conducted for each value of Δs.

For the regular Markov chain, the extinction was shown to be certain if, and only if, the mean number of offspring per individual did not exceed one (see for example [5]). This means in our case that the process of correlated fibre breaks eventually becomes extinct if, and only if, the mean number of fibres, broken due to one fibre break, does not exceed one. Therefore, we can use the condition

$$N_1(s_c) = 1 \tag{7}$$

as a condition that determines the critical stress. Below s_c the process of correlated fibre breaks eventually becomes extinct and a composite layer as a whole keeps its carrying capacity. If the acting stress exceeds s_c, a fibre break with a certain probability causes an avalanche of correlated fibre breaks and, therefore, a composite macrofracture. This probability increases with the increase of applied stress and the number of potential fracture sites. The fracture of a composite in cases $> s_c$ is considered in more detail in the appendix.

Use Equation 7 in order to obtain an analytical expression for s_c. Experiments show that a real composite fails at a comparatively low level of fibre fractures, which means that $F(s)$ is small in comparison with one at the fracture stress. Therefore, we can substitute the exponent function in the Weibull distribution by two first terms of its expansion about zero according to Taylor's theorem. Thus we have $F(s) = a(s/s_0)^b$ and hence

$$N_1 = N_0 a[(s + \Delta s)^b - s^b]/s_0^b \tag{8}$$

Further, we use binomial expansion for $(s + \Delta s)^b$, and because $\Delta s \ll s$ we delete in the expansion terms having second and higher powers of Δs; thus we take

$$(s + \Delta s)^b = s^b + bs^{(b-1)}\Delta s$$

Substituting this into Equation 8 we have $N_1 = (1/s_0)N_0 ab(s/s_0)^{b-1}\Delta s$.

$$N_1 = ab(s_c/s_0)^b = 1 \tag{9}$$

One can see that Equation 9 for critical stress, which follows from the branching model, coincides with Equation 4, which is used for calculating a composite critical stress in Rosen's model of delocalized fracture. However, there is a difference between the models. The Rosen model of a delocalized fracture is based on an evaluation of maximal carrying capacity of the fibre bundle and for that model Equation 4 is considered as precise, regardless of the size of a bundle. In the case of the branching model considered here, Equation 9 is an approximate solution and can be considered as precise only in the limiting case of a bundle consisting of an infinitely large number of fibres. This difference arises from the non-linear effects related to microfracture events.

2.1.2. Non-linear effects

In order to elicit the nature of non-linear effects at microfractures, we investigate the case of a fracture of a bundle with a finite number of fibres in more detail. The number of broken fibres in a bundle produced by one fibre break is given by Equation 5. Using Equation 7 we can obtain an equation determining critical stress for a branching process of correlated fibre breaks in a bundle of any size

$$F(s_c + \Delta s) - F(s_c) = 1/N_0 \tag{10}$$

where $\Delta s = s_c/[N_0(s_c)]$.

Equation 10 is a transcendental one and can be solved only numerically. Nevertheless, some characteristic features of the solution can be deduced from the analytical form of the equation. One can observe that the equation incorporates the size of a bundle as a parameter. This means that the fracture stress of a fibre bundle depends on the size of the bundle. This effect does not follow from classical models of composite fracture and is described only on the basis of the generalized model developed in the present paper.

The dependence of a fracture stress of a fibre bundle upon the bundle size arises out of the non-linear dependence of the probability of microfractures upon overstressing caused by other microfractures. In a case where the number of fibres in a bundle is large enough, we can consider all increments related to overstressing due to one fibre break as linear functions. This means, for instance, that if we double the size of the bundle, the number of fibres that potentially can be broken increases two-fold, but the overstressing due to one fibre break decreases two-fold; also, the probability of fibre fracture as a consequence of additional overstressing decreases two-fold. Therefore, the factors compensate each other. The situation is different when the total number of fibres in a bundle is small. In this case, if we, for example, decrease the size of a bundle by 50%, the number of potential fracture sites also will be 50% less. The overstressing per fibre in the bundle due to one fibre break will increase two-fold but the increment of probability of fibre fracture due to overstressing will be more than two-fold. This means that the probability of additional fracture due to one fibre break is higher for small bundles.

Mathematically, we can describe this situation in the following way: In the case of $N_0 \rightarrow \infty$ we can replace the exponent in the Weibull distribution by the two first terms of the expansion of Weibull distribution over $s = 0$ according to Taylor's theorem, namely

$$F(s + \Delta s) = F(s) + \Delta s \mathrm{d}F(s)/\mathrm{d}s \qquad (11)$$

Taking $\Delta s = 1/[N_0(1 - F(s))]$, and substituting Equation 11 into Equation 10 we have

$$N_1 = N_0 \Delta s \mathrm{d}F(s)/\mathrm{d}s = [1/(1 - F(s))]\,\mathrm{d}F(s)/\mathrm{d}s$$

One can see that in this case the number of fibres broken in the first generation of a branching process does not depend on the size of the bundle. However, if we take into account the higher terms of Taylor's expansion, we obtain

$$F(s + \Delta s) = F(s) + \Delta s\,\mathrm{d}F(s)/\mathrm{d}s + 1/2\,\Delta s^2\,d^2F(s)/\mathrm{d}s^2$$

and therefore

$$N_1 = [1/(1 - F(s))]\,\mathrm{d}F(s)/\mathrm{d}s + 1/[N_0(1 - F(s))]^2\,\mathrm{d}^2F(s)/\mathrm{d}s^2$$

The presence of N_0 in the denominator of the second term in the last equation shows that, in the case of variation of the size of a small bundle, the increase of overstressing due to one fibre break is not compensated by the decrease of the number of fibres that can be potentially broken.

One can solve Equation 10 using numerical methods. In Fig. 1, an example of a solution obtained by the use of MathCAD software facilities, is shown. The critical stress for a fibre bundle, a function of a bundle size, is calculated for cases of wide (parameter b in the Weibull distribution is equal to 4.6) and narrow ($b = 11$) distributions of fibre strength. (The other parameters used in Weibull distribution were $s_0 = 4320$ MPa, $a = 0.25$ in both cases.) The critical stress of the bundle is given in units of the critical stress calculated according to Rosen's model from Equation 4. One can see from curves 1 and 2 in Fig. 1 that the critical stress of a bundle consisting of a finite number of fibres is lower than the critical stress of an infinitely large bundle and increases asymptotically with the increase of number of fibres to the maximal value. Also, one can see from Fig. 1 that the decrease in strength with the decrease of the number of elements in the bundle is higher for bundles consisting of more homogeneous elements.

The dependence of composite strength upon the number of structural units constituting the composite is important for the composites with internal substructure, nanocomposites and multilayers of thin films.

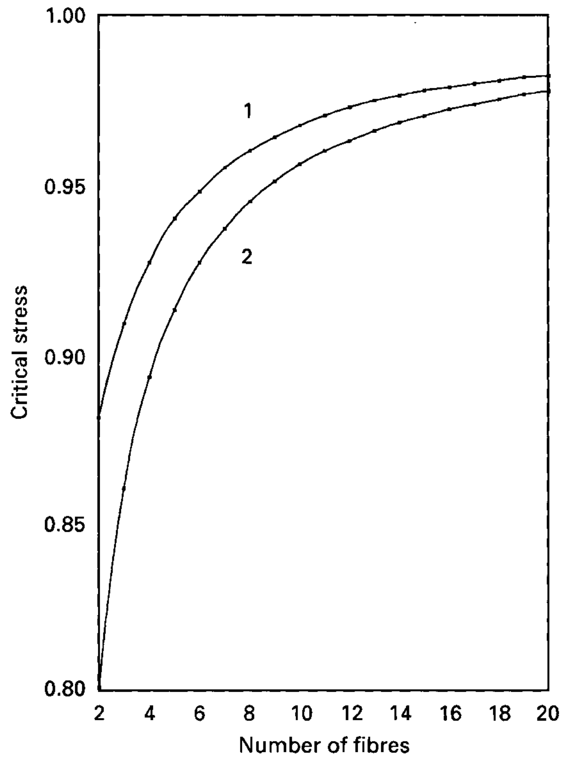

Figure 1 The dependence of the critical stress of a composite upon the number of fibres in the composite cross-section (in units of critical stress of a composite with an infinitely large cross-section). Variation of fibre strength was described by a Weibull distribution with parameter $b = 4.6$ (curve 1) and $b = 11$ (curve 2).

2.2. Model of localized fracture

Models of localized fracture are based on the suggestion that, after a break of a fibre, only fibres close to the broken one experience overstressing. Local overstressing is evaluated on the basis of one or another model of stress transfer from the broken fibre to the unbroken fibres. Then, the probability of the initiation of groups of two, three, four, etc., fibres due to overstressing is evaluated. Considering these probabilities as a function of applied stress, the probability of the failure of the layer $FF(s)$ is computed. The value s, at which $FF(s)$ reaches a certain magnitude, for instance $FF(s) = 0.5$, is usually taken as the macrofracture stress.

Consider the model of localized fracture from the viewpoint of a branching process. According to the basic model assumptions, whenever a fibre is broken, n fibres adjacent to the broken one experience overstressing. Assume here that the overstressing, Δs, is the same for all n fibres. (In the case of different Δs a consideration similar to the following can be conducted for each value of Δs.)

Using the same simplifying assumptions as in the previous case, one can find that for the first generation of the branching process the number of fibre breaks initiated by the first broken fibre is

$$n_1 = n[F(s + \Delta s) - F(s)] = n(a/s_0)(s/s_0)^{(b-1)}\Delta s \qquad (12)$$

Taking, as it is usually supposed for local overstressing, $\Delta s = (p - 1)s$, where p is the coefficient of local overstressing, we obtain

$$n_1 = n(p-1)ab(s/s_0)^b \qquad (13)$$

where p_0 can be called the effective coefficient of local overstressing. Also, as in the previous case, the condition in Equation 7 can be used to obtain the critical stress. Therefore, from the condition $n_1 = 1$ we have

$$s_0 = [n(p-1)]^{(-1/b)} s_0 (ab)^{(-1/b)} \qquad (14)$$

One can see that with the exception of the coefficients characterizing the local overstressing (n, p), the other terms in Equation 14 coincide with Equation 9 determining the critical stress for a delocalized fracture in a linear case. In a non-linear case a numerical solution of Equation 10 should be determined.

From the viewpoint of the branching process, the location of overstressed fibres is not important. They can be adjacent to a broken fibre or distant from the fracture site; therefore one can refer to Fig. 1 in order to understand the qualitative behavior of the possible non-linear solutions, also, in the case of localized fracture. In the last case instead of the total number of fibres in the composite cross-section, N_0, the number of overstressed fibres surrounding the broken fibre, n, should be used.

One can see from Equation 14 that the effect of overstressing on the composite strength in a linear approximation is proportional to the overstressing coefficient in $-1/b$ power. This means that in linear approximation the overstressing effect is essential only for composites fabricated from fibres having a large variation of properties, characterized by small b ($b \sim 3, \ldots, 4$). In the case of homogeneous fibres, ($b \sim 6, \ldots, 12$ or more), the local overstressing in a linear approximation has very little influence on the composite strength. This result is in compliance with experimental data given in [6, 7]. In [6] it was shown that in Kevlar-epoxy composites ($b = 10.2$ for Kevlar fibres) the fracture of a fibre does not cause failure of adjacent fibres, the fibre breaks occur in a delocalized way and "the failed samples have a brush-like appearance". In [7] composites based on high perormance polyethylene (HP–PE) fibres ($b = 8.5$) were studied. It was shown that "in untreated HP-PE epoxy composites fibres failed independently from each other and did not interact".

The situation is different if we consider the same process from the viewpoint of non-linear theory. The fracture stress decrement due to the non-linear effect is more essential for fibres with a small variation of strength (see Fig. 1).

The transition from the localized fracture mode to the delocalized fracture mode can be achieved due to control over the composite structure (properties of the interface) provided by modern technologies for advanced composites fabrication.

3. Discussion

We discuss how essential is the influence of model assumptions upon the obtained results. Actually, we made one simplifying assumption: we supposed that the overstressing will be the same for all generations of a branching process. Due to this assumption, the chain process can be considered as a regular Markov chain. In reality, the overstressing increases from one generation to the next. In the case where Δs is different for the different generations of the branching process, one can imagine a situation where the branching process is substantial in the first generation, and becomes overcritical in the second or higher generations. In most cases, the exact solutions for varying Δs can be obtained only numerically, and throughout investigation of these cases, it is a subject of separate research. Nevertheless, here we can roughly evaluate the qualitative difference between the solution for the branching process in the cases where (i) Δs is constant and (ii) Δs increases from generation to generation.

For the Markov process with constant increments, s_c is to be considered as a threshold value: at stresses below s_c fracture does not occur, at values higher than s_c fracture happens with a probability that tends to one with increasing stress and the number of potential microfracture sites (see the appendix).

In the case of Δs increasing from generation to generation, with a certain probability the macrofracture may start at a stress lower than the value s_c, determined by Equation 10. Therefore, s_c should be considered as an average value of strength with probable variations to smaller as well as higher values. The dispersion of the strength of samples should be determined numerically for every particular model of stress redistribution. One can also expect that in most cases, the difference between the exact numerical solution and approximate analytical solution obtained here will be small. The suggestion follows from the fact that transition from the subcritical branching process to the overcritical one can only happen within a small range over the threshold value of the stress. The last conclusion follows from two characteristic features of the branching process considered.

1. An increment of acting stress, Δs_{k+1}, at transition from the kth generation of the branching process to the $k+1$ generation is small in comparison with both the acting stress and the critical stress ($\Delta s = s/N$).
2. For the subcritical branching process ($N_1 < 1$) the probability of the process surviving in the rth generation ($r = 1, 2, \ldots$) decreases exponentially with increasing r (see [3]).

Therefore, the probability of a deviation from the fracture stress calculated according to Equation 10 decreases exponentially with the magnitude of the deviation. Besides, in the last case the difference between the critical stress calculated from Equation 10 and the actual critical stress can be $r\Delta s$. Taking into account that $\Delta s = s/N$ one can obtain $r\Delta s = rs/N \ll s$ in almost every practically important case.

This also is in compliance with the fact that for the delocalized fracture the approximation of a branching process by a branching process with constant overstressing gives the critical stress that coincides with the precise solution obtained earlier by Rosen for a composite with an infinitely large cross-section.

4. Conclusions

In the present paper we introduce a new criterion for the fracture of composite materials reinforced with continuous fibres. This criterion can be generalized for the case of fracture of a composite with any structure, provided the fracture occurs due to overstressing of the elements of the composite at microfractures. It can be especially important for advanced materials, such as ceramics, nanocomposites and multilayers. In our consideration we started from the apparent difference between the two groups of models traditionally used for describing fracture of fibre composites, namely the models of localized and delocalized fracture. Then we showed that models of both groups can be mathematically described on the basis of the same branching process.

The use of a generalized model allowed us to obtain a fracture criterion common to models of localized and delocalized fracture. This result became possible because the mathematical formulae describing the branching processes did not depend upon the mutual location of fibres, the difference between the cases of localized and delocalized fracture being only in the level of overstressing and the number of fibres experiencing that overstressing. Because the basic principles govering branching processes are the same for any model, the functional form of the final expression evaluating the critical stress is also the same. The difference is in the numerical coefficients that depend on the simplifying assumptions used for calculation of the number of overstressed fibres and the level of overstressing.

The use of a fracture criterion, which follows from the theory of branching processes, allows us to obtain a new result that previously has not been considered theoretically. This result consists of the dependence of the critical stress of a composite upon the size of the cross-section of the composite. This dependence, as was shown, can be explained as a non-linear effect. This effect arises as a consequence of a non-linear increase of fibre break probability due to overstressing caused by other microfractures. This result is essential for evaluation of the strength of real construction elements, which in many cases are reinforced with a limited number of fibres (or fibre layers) due to size or weight requirements. The non-linear effect, also, is essential for modelling the mechanical properties of structured composites. In this case structural units of the composite consist of a finite number of elements and non-linear theory should be used for the prediction of the strength of the structural elements and, therefore, of the entire composite.

Appendix

In the case where the mean number of fibre breaks caused by overstressing due to another fibre break is more than one, the probability of an extinction of the branching process of correlated fibre breaks is determined as the smallest positive root of the equation

$$f(z) = z$$

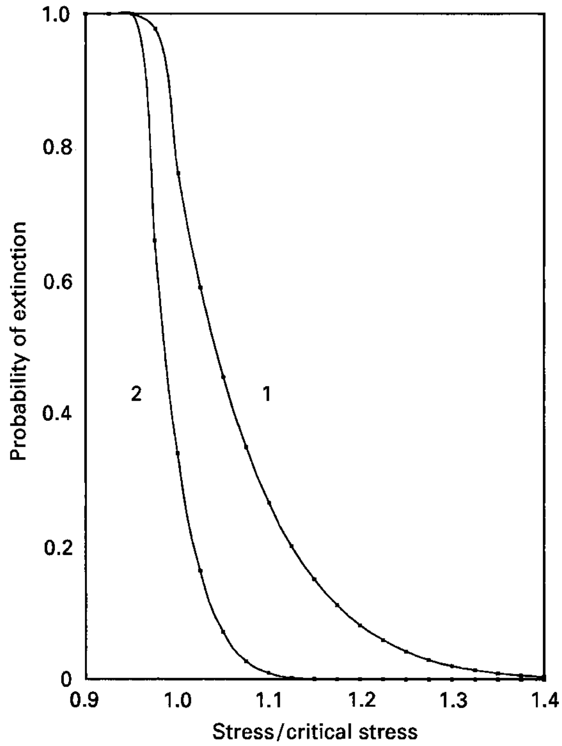

Figure A1 Probability of extinction of the process of correlated fibre breaks as a function of acting stress (in units of critical stress for an infinitely large composite sample). Variation of fibre strength was described by the Weibull distribution with parameter $b = 4.6$ (curve 1) and $b = 11$ (curve 2).

where $f(z)$ is the probability generating function

$$f(z) = \sum_k p_k z^k$$

where p_k, in its turn, is the probability that the fibre fracture causes k new fibre breaks.

In the considered model, fibre breaks in the ith generation are independent from each other; therefore the binomial distribution can be used for p_k

$$p_k = \frac{N!}{n!(N-n)!} G^n(s)[1 - G(s)]^{N-n}$$

where $G = [F(s + \Delta s) - F(s)]/[1 - F(s)]$, $\Delta s = s/N$.

It has been shown [5] that in this case the probability generating function is

$$f(z) = [1 - G(s) + zG(s)]^N$$

and, therefore, the probability of extinction, z_e, in this case is determined by the equation

$$[1 - G(s)(1 - z_e)]^N = z_e$$

In Fig. A1 the numerical calculations for probability of extinction as a function of acting stress are shown. The cross-section of the composite was assumed to consist of 15 fibres, and the fibres' strength variation was described by the Weibull distribution with parameters $b = 4.6$ (curve 1) and $b = 11$ (curve 2), $s_0 = 4320$ MPa, $a = 0.25$.

One can see from Fig. A1 that the probability for the composite to survive falls rapidly at stresses above the critical stress.

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